Gravity-balancing of spatial robotic manipulators

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ABSTRACT

Gravity balancing is often used in industrial machines to decrease the required actuator efforts during motion. In the literature, a number of methods have been proposed for gravity balancing that include counterweights, springs, and auxiliary parallelograms that determine the center of mass. This paper describes the underlying theory of gravity balanced spatial robotic manipulators through a hybrid strategy which uses springs in addition to identification of the center of mass using auxiliary parallelograms.

In the literature, it has been proved that the fixed end springs alone can not gravity balance a spatial multi-degree-of-freedom mechanism unless the attachment points of the springs are actively changed during motion of the manipulator. A significant contribution of this paper is to show that springs with fixed ends are sufficient to gravity balance a spatial mechanism if the hybrid method of gravity balancing is used where the center of mass is identified first through auxiliary parallelograms. Also, the system remains gravity balanced even if the orientation of the base is changed, i.e., the direction of the gravity is changed with respect to the base. We present the method for gravity compensation of two and three degrees-of-freedom (DOF) spatial manipulators. A prototype with the underlying principles of this paper was fabricated at the University of Delaware.

1 INTRODUCTION

A machine is said to be gravity balanced if joint actuator inputs are not needed to keep the system in equilibrium at any configuration of the machine. The system essentially behaves as if its motion is in a gravity-less environment. Over the years, gravity-balanced machines have been realized through clever designs using counterweights, springs, and auxiliary parallelograms ([7], [8], [6], [5], [2], [4], [9], [3], [1]). A number of mathematical descriptions can be given for gravity balanced machines such as (i) system center of mass remains inertially fixed during motion; (ii) potential energy remains invariant with configuration of the system; (iii) system has counter masses that balance the machine in every configuration.
These mathematical conditions have been physically realized through clever engineering such as: (a) Countermass on each body of the machine is used to inertially fix the center of mass of the system, (b) Springs are used at appropriate places in the machine such that the sum total of the gravitational potential energy and spring potential energy together becomes invariant with configuration, (c) auxiliary parallelograms based on knowledge of geometry and inertia property are used to physically identify the center of mass of the machine. While each of these methods have their scientific core along with advantages and disadvantages, this paper focuses on a hybrid methodology for gravity balancing of spatial robotic manipulators that combines the underlying fundamentals between these different methods.

In the methods presented in the literature using springs, gravity balancing is achieved for a fixed direction of the gravity vector. However, in the methods presented in this paper, balancing is independent of the direction of the gravity vector. In other words, the base of the machine can be oriented in any direction with respect to the direction of the gravity vector. This paper uses the notion of auxiliary parallelograms to first locate the center of mass. In addition, springs are connected through this point to make the potential energy invariant with configuration. For these reasons, this hybrid two-step approach is desirable over approaches which do not consider finding the center of mass first.

At this time, only a few studies exist on gravity balancing of spatial manipulators. A spatial two-dof serial manipulator was studied for gravity balancing in [10]. The authors proved in this study that for a two-DOF manipulator, the conditions of gravity balancing requires that the end points of the springs move relative to the mechanism, i.e., be independently actuated. This will require extra actuators to be mounted on the system and is undesirable. In this paper, we show that there is no need for the end points of the springs to be actively moved if the hybrid method presented in this paper is used.

The organization of this paper is as follows: Section 2 describes the underlying theory of gravity balanced robotic manipulators through hybrid strategy which uses springs in addition to identification of the center of mass using auxiliary parallelograms. Next, we present the method for gravity compensation of two and three degrees of freedom (DOF) spatial robotic manipulators.

2 GRAVITY-BALANCING: A HYBRID METHOD

The underlying idea behind the hybrid method is as follows: (i) Locate the center of mass of the manipulator; (ii) Select springs to connect to the center of mass such that the total potential energy of the system is invariant with configuration. This method allows one to physically locate the center of mass of the manipulator and connect this point to the inertially fixed frame through springs.
2.1 Location of System Center of Mass

Consider an n-link articulated serial manipulator with revolute joints. The Denavit-Hartenberg (DH) parameters for two successive links, i.e., $\theta_i$, $d_i$, $\alpha_i$ and $a_i$, shown in Fig. 1, which characterizes the motion of link $i$ with respect to link $i - 1$ in terms of the following transformation matrix.

$$
i^{-1}T_i =
\begin{pmatrix}
\cos \theta_i & -\cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & a_i \cos \theta_i \\
\sin \theta_i & \cos \alpha_i \cos \theta_i & -\sin \alpha_i \cos \theta_i & a_i \sin \theta_i \\
0 & \sin \alpha_i & \cos \alpha_i & d_i \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

Here, $i = 1, \ldots, n$ and $\theta_i$ is the variable joint angle, while all other DH parameters are constants. The location of the system center of mass $C$ for this manipulator from the reference point (origin) $O$ is

$$
r_{OC} = \frac{1}{M} \sum_{i=1}^{n} m_i r_{OC_i},
$$

where

$$
M = \sum_{i=1}^{n} m_i.
$$

Here, $M$ is the total mass of the manipulator, $m_i$ is the mass of link $i$, and $r_{OC_i}$ is the location of the center of mass of link $i$, $C_i$, from point $O$. The vector $r_{OC_i}$ can be written as

$$
r_{OC_i} = r_{OO_i} + r_{O_iC_i},
$$

where $O_i$ is the origin of the frame attached to link $i$. Using Fig. 1, $r_{OO_i}$ is

$$
r_{OO_i} = \sum_{j=1}^{i} (d_j z_{j-1} + a_j x_j)
$$
and the location of center of mass of link \( i \) from point \( O_i \) in its local frame is expressed as

\[
r_{O_iC_i} = \beta_{ix}x_i + \beta_{iy}y_i + \beta_{iz}z_i
\]  

(6)

Upon substitution of Eqs. (5) and (6) into Eq. (4) and inserting the result thus obtained into Eq. (2), we obtain

\[
r_{OC} = \frac{1}{M} \sum_{i=1}^{n} m_i \left[ \sum_{j=1}^{i} (d_j z_{j-1} + a_j x_j) + \beta_{ix}x_i + \beta_{iy}y_i + \beta_{iz}z_i \right]
\]  

(7)

or in compact form as

\[
r_{OC} = \sum_{i=1}^{n} (\gamma_{ix}x_i + \gamma_{iy}y_i + \gamma_{iz}z_i).
\]  

(8)

Here, it is assumed \( d_1 = 0 \). Note that \( \gamma_{ix}, \gamma_{iy}, \gamma_{iz} \) are factors of geometry and mass distribution of the links and are usually denoted by the terminology “scaled lengths”. Vector \( r_{OC} \) can be expressed in any coordinate frame by expressing the vectors \( x_i, y_i \) and \( z_i \) in this coordinate frame. For example, in the inertially fixed reference frame can be written as

\[
r_{OC} = \delta_1 x_0 + \delta_2 y_0 + \delta_3 z_0.
\]  

(9)

where \( \delta_i, i = 1, 2, 3 \) are configuration dependent, functions of scaled lengths, and also DH parameters of all links.

### 2.2 Springs Selection

Upon locating the system center of mass, the potential energy of the system due to the gravity can be written as

\[
V_g = Mg g \cdot r_{OC}
\]  

(10)
where \( \mathbf{g} \) is the unit vector along the gravity. It can be expressed in the inertial frame as

\[
\mathbf{g} = \xi_1 \mathbf{x}_0 + \xi_2 \mathbf{y}_0 + \xi_3 \mathbf{z}_0
\]  

where all \( \xi_i, i = 1, 2, 3 \) are constants.

For gravity compensation, we attach a spring of stiffness \( k \) between the system center of mass \( \mathbf{C} \) and a point \( P \) along the gravity direction at a distance \( d \) from origin, \( O \) shown in Fig 2. The strain potential energy due to the spring is

\[
V_s = \frac{1}{2} k x^2
\]  

where \( x^2 \) is

\[
x^2 = \|\overrightarrow{PC}\| \cdot \|\overrightarrow{PC}\|
\]  

Here, \( \overrightarrow{PC} \) can be written as

\[
\overrightarrow{PC} = \mathbf{r}_{OC} - \mathbf{r}_{OP}
\]  

and

\[
\mathbf{r}_{OP} = d(\xi_1 \mathbf{x}_0 + \xi_2 \mathbf{y}_0 + \xi_3 \mathbf{z}_0)
\]  

Upon substitution of Eqs.(15) and (9) into Eq.(14) and inserting the result thus obtained into Eq.(13) and then substituting this into Eq.(12), we obtain

\[
V_s = \frac{1}{2} k [(\delta_1 - d\xi_1)^2 + (\delta_2 - d\xi_2)^2 + (\delta_3 - d\xi_3)^2]
\]  

Upon substitution of Eqs.(11) and (9) into Eq.(10) and expanding the result thus obtained, we get \( V_g \) as

\[
V_g = Mg(\delta_1 \xi_1 + \delta_2 \xi_2 + \delta_3 \xi_3)
\]  

Therefore, the total potential energy of the system is the sum of potential energies due to gravity and the spring

\[
V = V_g + V_s
\]

\[
= (Mg - kd)\delta_1 \xi_1 + (Mg - kd)\delta_2 \xi_2 + (Mg - kd)\delta_3 \xi_3
\]

\[
+ \frac{1}{2} k (\xi_1^2 + \xi_2^2 + \xi_3^2)d^2 + \frac{1}{2} k (\delta_1^2 + \delta_2^2 + \delta_3^2)
\]  

The total potential energy is invariant with configuration if the coefficients of configuration variable terms vanish. Hence, the stiffness of spring \( k \) is determined by setting the coefficients of \( \delta_i \xi_i, i = 1, 2, 3 \) to zero, i.e.,

\[
k = \frac{Mg}{d}.
\]  

The only configuration varying term remaining in the total potential energy is \( \frac{1}{2} k (\xi_1^2 + \xi_2^2 + \xi_3^2) \) or simply \( \frac{1}{2} k \mathbf{r}_{OC} \cdot \mathbf{r}_{OC} \). It may be noted that this term is function of all joint
angles, i.e., \( \theta_i, \; i = 1, \ldots, n \). The strategy is to add appropriate springs to the system such that the total potential energy of the system becomes configuration invariant.

If we change the base of the manipulator with respect to the gravity vector, the unit gravity vector \( \mathbf{g} \) can be re-expressed in the inertial frame as

\[
\mathbf{g} = \xi'_1 \mathbf{x}_0 + \xi'_2 \mathbf{y}_0 + \xi'_3 \mathbf{z}_0,
\]

where all \( \xi'_i, \; i = 1, 2, 3 \) are constants. This is the only change that we should make when the base has a different orientation with respect to the gravity vector. It is clearly seen from the procedure mentioned in this section that the gravity balancing of the manipulator does not depend on the coefficients \( \xi'_i, \; i = 1, 2, 3 \). Therefore, this method results in gravity balancing for all directions of the gravity vector. In other words, the base of machine can be installed in any direction with respect to gravity vector.

We will apply this method of making the total potential energy invariant with configuration to specific examples in the next section.

3 EXAMPLES

3.1 Two DOF Robotic Manipulator

Consider a two DOF spatial manipulator, i.e., \( n = 2 \) in Fig. 2. Here, the axis of joint 1 (\( \mathbf{z}_0 \)) is not parallel to the axis of joint 2 (\( \mathbf{z}_1 \)), i.e., \( \alpha_1 \neq 0 \). However, we assume \( \mathbf{z}_1 \) is parallel to the axis of joint 3 (\( \mathbf{z}_2 \)), i.e., \( \alpha_2 = 0 \). The location of the system center of mass of two DOF manipulator can be derived using \( n = 2 \) in Eq.(8) as follows:

\[
\mathbf{r}_{OC} = \sum_{i=1}^{2} (\gamma_{ix} \mathbf{x}_i + \gamma_{iy} \mathbf{y}_i + \gamma_{iz} \mathbf{z}_i),
\]

where the coefficients are constants and can be computed inserting \( n = 2 \) into Eq.(7). The resulting coefficients are

\[
\begin{align*}
\gamma_{1x} &= a_1 + \frac{m_1}{M} \beta_{1x} \\
\gamma_{1y} &= \frac{m_1}{M} \beta_{1y} \\
\gamma_{1z} &= \frac{m_1}{M} \beta_{1z} + \frac{m_2}{M} d_2 \\
\gamma_{2x} &= \frac{m_2}{M} (a_2 + \beta_{2x}) \\
\gamma_{2y} &= \frac{m_2}{M} \beta_{2y} \\
\gamma_{2z} &= \frac{m_2}{M} \beta_{2z}
\end{align*}
\]
The expression $\frac{1}{2}k\mathbf{r}_{OC} \cdot \mathbf{r}_{OC}$ for two-DOF manipulator is

$$\frac{1}{2}k\mathbf{r}_{OC} \cdot \mathbf{r}_{OC} = k(\gamma_{1x}\gamma_{2x}\mathbf{x}_1 \cdot \mathbf{x}_2 + \gamma_{1x}\gamma_{2y}\mathbf{y}_1 \cdot \mathbf{y}_2 + 
\gamma_{1y}\gamma_{2x}\mathbf{x}_1 \cdot \mathbf{z}_2 + \gamma_{1y}\gamma_{2y}\mathbf{y}_1 \cdot \mathbf{z}_2 + 
\gamma_{1z}\gamma_{2x}\mathbf{z}_1 \cdot \mathbf{x}_2 + \gamma_{1z}\gamma_{2y}\mathbf{z}_1 \cdot \mathbf{y}_2 + 
\gamma_{1z}\gamma_{2z}\mathbf{z}_1 \cdot \mathbf{z}_2)$$

Upon substitution of $i = 2$, $\alpha_2 = 0$ in the transformation matrix given by Eq.(1), we obtain the angle between the axes of the local frames of two successive links 1 and 2 shown in Table 1. Then, inserting the results from Table 1 into Eq.(28), one obtains the nonzero configuration varying terms in $\frac{1}{2}k\mathbf{r}_{OC} \cdot \mathbf{r}_{OC}$. These terms are $k(\gamma_{1x}\gamma_{2x} + \gamma_{1y}\gamma_{2y})\cos \theta_2$ and $k(-\gamma_{1x}\gamma_{2y} + \gamma_{1y}\gamma_{2x})\sin \theta_2$. We add two springs to compensate for the two configuration varying terms: a spring with stiffness $k_1$ between the axes $\mathbf{x}_1$ and $\mathbf{x}_2$ and a second spring $k_2$ between the axes $\mathbf{y}_1$ and $\mathbf{x}_2$. The connection points $d_i'$, $i = 1, \ldots, 4$ are shown in Fig. 3(a). The values of $k_1$ and $k_2$ are selected such that the potential energy becomes invariant. The expressions for these two stiffnesses are

$$k_1 = \frac{k(\gamma_{1x}\gamma_{2x} + \gamma_{1y}\gamma_{2y})}{d_1'd_2'}$$

$$k_2 = \frac{k(-\gamma_{1x}\gamma_{2y} + \gamma_{1y}\gamma_{2x})}{d_3'd_4'}$$
Table 1: The angle between the axes of the local frames at links 1 and 2

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<tr>
<td>z_2</td>
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Therefore, the two DOF spatial manipulator is completely gravity balanced by using three springs shown in Fig. 3(b). These three springs are: (i) a spring with stiffness \( k = \frac{M_g}{d} \) from the center of mass of the manipulator to a point on a line from the origin of the inertially fixed frame parallel to the gravity vector, (ii) two more springs with stiffnesses \( k_1 \) and \( k_2 \). Please note that the location of the center of mass of the manipulator can be identified using auxiliary parallelograms based on Eq.(21). However, the location of the center of mass has a simpler expression when described with respect to point \( O_1 \) instead of \( O \) and is given as

\[
r_{O_1C} = r_{OC} - r_{OO_1} = r_{OC} - a_1 x_1,
\]

where \( r_{OC} \) was defined in Eq.(21).

### 3.1.1 Two DOF Manipulator: Special Case

We obtain the conditions for gravity balancing of a 2-link spatial manipulator when the center of mass of the links are located on \( x_i z_i \) plane, i.e., \( \gamma_{iy} = 0 \) in Eq.(21). Using these simplifications in Eqs.(29) and (30), one gets

\[
k_1 = \frac{k \gamma_{1x} \gamma_{2x}}{d_1' d_2'} \quad \text{(32)}
\]

\[
k_2 = 0. \quad \text{(33)}
\]

Therefore, the 2-link spatial manipulator is gravity balanced using only two springs - one from the center of mass of the manipulator to an inertially fixed point along the gravity direction passing through the origin, with stiffness \( k = \frac{M_g}{d} \) and the second spring of stiffness \( k_1 \) between the axis \( x_1 \) and \( x_2 \), shown in Fig. 4(a).

### 3.1.2 Prototype of Two DOF Manipulator

An engineering prototype based on the design explained in Section 3.1.1 was fabricated at University of Delaware and is shown in Fig. 4(b). This prototype is made of wooden and aluminum parts. It verifies the conceptual idea of spatial gravity balancing of two DOF manipulator and was tested on different configurations of end-effector and also different orientations of the base (See http://mechsyst4.me.udel.edu/movies/rehrob/spabalmov/).
Figure 4: (a) 2-link gravity balanced spatial manipulator with spring connections: Special case, (b) Photographs of prototype of the two DOF spatial gravity balanced manipulator.

### 3.2 Three DOF Spatial Manipulator

Consider a three DOF spatial manipulator, i.e., $n = 3$, as shown in Fig. 5. Here, the three joint axes $1, 2, \text{ and } 3$ axes are not parallel to each other. However, we assume that joint axis $3 (z_2)$ is parallel to joint axis $4 (z_3)$, i.e., $\alpha_3 = 0$. The location of the system center of mass of three DOF manipulator can be derived using $n = 3$ in Eq.(8) as follows:

$$
\mathbf{r}_{OC} = \sum_{i=1}^{3} (\gamma_{ix}\mathbf{x}_i + \gamma_{iy}\mathbf{y}_i + \gamma_{iz}\mathbf{z}_i),
$$

where the coefficients are constants and can be computed by inserting $n = 3$ into Eq.(7). Here, we assume the center of mass of the links are located on $x_i z_i$ plane, i.e., $\gamma_{iy} = 0$ in Eq.(21). The nonzero terms of expression $\frac{1}{2}\mathbf{kr}_{OC} \cdot \mathbf{r}_{OC}$ for the three-DOF manipulator are:

$$
k_{\gamma_{1x}\gamma_{2x}\mathbf{x}_1 \cdot \mathbf{x}_2}, k_{\gamma_{1x}\gamma_{2x}\mathbf{x}_1 \cdot \mathbf{z}_2}, k_{\gamma_{1x}\gamma_{2x}\mathbf{z}_1 \cdot \mathbf{x}_2}, k_{\gamma_{1x}\gamma_{2x}\mathbf{z}_1 \cdot \mathbf{z}_2}, k_{\gamma_{2x}\gamma_{3x}\mathbf{x}_2 \cdot \mathbf{x}_3}, k_{\gamma_{2x}\gamma_{3x}\mathbf{x}_2 \cdot \mathbf{z}_3}, k_{\gamma_{2x}\gamma_{3x}\mathbf{z}_2 \cdot \mathbf{x}_3}, k_{\gamma_{2x}\gamma_{3x}\mathbf{z}_2 \cdot \mathbf{z}_3}, k_{\gamma_{1x}\gamma_{3x}\mathbf{x}_1 \cdot \mathbf{x}_3}, k_{\gamma_{1x}\gamma_{3x}\mathbf{x}_1 \cdot \mathbf{z}_3}, k_{\gamma_{1x}\gamma_{3x}\mathbf{z}_1 \cdot \mathbf{x}_3}, k_{\gamma_{1x}\gamma_{3x}\mathbf{z}_1 \cdot \mathbf{z}_3}.
$$

Upon substituting $i = 2, 3$ and $\alpha_3 = 0$ in transformation matrix given by Eq.(1), we obtain the angles between the different axes of the coordinate frames as shown in Table 2. Inserting the results from Table 2 into the nonzero terms of expression $\frac{1}{2}\mathbf{kr}_{OC} \cdot \mathbf{r}_{OC}$, one obtains the remaining nonzero terms to be as $k_{\gamma_{1x}\gamma_{2x}\cos \theta_2}, k_{\gamma_{1x}\gamma_{2x}\sin \alpha_2 \sin \theta_2}, k_{\gamma_{1x}\gamma_{3x}\mathbf{x}_1 \cdot \mathbf{x}_3}, k_{\gamma_{1x}\gamma_{3x}\sin \alpha_2 \sin \theta_3}$ and $k_{\gamma_{2x}\gamma_{3x}\cos \theta_3}$.

We add four springs to compensate for these five terms: (i) a spring $k_1$ between axis $\mathbf{x}_1$ and $\mathbf{x}_3$, (ii) a spring $k_2$ between $\mathbf{x}_2$ and $\mathbf{x}_3$, (iii) a spring $k_3$ between $\mathbf{y}_2$ and $\mathbf{x}_3$, and (iv)
Figure 5: A 3-link spatial manipulator

A spring $k_4$ between axis $\mathbf{x}_1$ and $\mathbf{x}_2$ with connection points $d'_i$, $i = 1, \ldots, 8$, respectively, as shown in Fig. 6(a). The values of $k_i$ for $i = 1, \ldots, 4$ are derived to make the potential energy invariant with configuration. The expressions for the spring stiffnesses are

\begin{align*}
  k_1 &= \frac{k_1 \gamma_2 \gamma_2 \cos \alpha_2}{d_3 d'_1 \sin \alpha_2} \quad (35) \\
  d'_2 &= \frac{k_1 \gamma_2 \gamma_3}{k_1 d'_1} \quad (36) \\
  k_2 &= \frac{k_2 \gamma_2 \gamma_3 + k_1 a_2 d'_1}{d_3 d'_4} \quad (37) \\
  k_3 &= \frac{k_1 \gamma_2 \gamma_3 \sin \alpha_2 + k_1 d'_2 d_3 \sin \alpha_2}{d_5 d'_6} \quad (38) \\
  k_4 &= \frac{k_1 \gamma_2 \gamma_2 - k_1 d'_1 a_2}{d'_4 d'_5} \quad (39)
\end{align*}

Note that all $d'_i$, $i = 1, \ldots, 8$, are arbitrary except $d'_2$, specified by Eq.(36).

Table 2: The dot products of the axes of the local coordinate frames

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The location of the system center of mass can be realized using the following procedure: (i) Locate the center of mass of the first two links $C_{12}$ using the method shown in Section 3.1; (ii) Locate the center of mass of the third link $C_3$; (iii) Locate the system center of mass $C$ using a fractional mechanism shown in Fig. 6(b). A fractional mechanism works in the following way: If the masses $m_1 + m_2$ and $m_3$ are located at points $C_{12}$ and $C_3$, respectively, we use two links of arbitrary lengths $l_1$ and $l_2$ connected by a revolute joint. Two auxiliary links of lengths $\eta_1 l_1 = \frac{m_1 + m_2}{m_1 + m_2 + m_3} l_1$ and $\eta_2 l_2 = \frac{m_3}{m_1 + m_2 + m_3} l_2$ are attached to the parent links to form a parallelogram with revolute joints so that all joints of the fractional mechanism are parallel to each other. $\eta_1 l_1$ and $\eta_2 l_2$ form a parallelogram and locate the system center of mass $C$ if this mechanism is attached to the manipulator at points $C_{12}$ and $C_3$ using two spherical joints. This method can be extended to include more links within the spatial manipulator.

4 CONCLUSION

The paper described the underlying theory of gravity balanced for spatial robotic manipulators through a hybrid strategy which uses springs in addition to identifying the center of mass using parallelograms and auxiliary links. In the literature, it has been proved that springs alone cannot gravity balance a spatial multi-degree-of-freedom mechanism unless the attachment points of the springs are actively changed during motion of the manipulator. This paper showed that springs with ends fixed on the mechanism are sufficient to gravity balance a spatial mechanism if a hybrid method for gravity balancing is used, where the center of mass is identified first through auxiliary parallelograms. Also, the system remains gravity balanced even if the orientation of the base is changed, i.e., the direction of the gravity is changed with respect to the base. The method for gravity compensation was illustrated by two and three degrees-of-freedom (DOF) spatial ma-
nipulators. Currently, a prototype with the underlying principles of this paper is under fabrication.

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**References**


