ABSTRACT

Stewart platform is a good candidate for high speed and six degree of freedom motion control, finding its application in the area of multi-axis vibration control. The critical problems associated with Stewart platform are the high non-linearity and uncertainty in its dynamics. To model the system dynamics of the Stewart platform a simple linear model is developed, for the motion of the Stewart platform in stationary position using Newton-Euler equations of motion for rigid body. The damping and stiffness matrix are found to be proportional to each other and so simplifying the dynamics. For the developed dynamic model various control strategies are applied and the performance of the system is studied to identify the best suited control strategy for vibration isolation applications. Modeling and testing of the control strategies are done using MATLAB and is to be verified experimentally.

INTRODUCTION

Control of vibration has received a great deal of interest in all precision pointing and positioning systems. Control of vibration is achieved by both passive and active means. While passive means are effective in unpowered configurations introducing no risk of instability. Active means are found to be well suited for dynamic systems, promising increased isolation performance. In general, the requirements for vibration control in precision applications maybe classified into two levels, vibration isolation[1] at a component level and structural vibration suppression at a system level. Vibration isolation at component level would be referred to as active vibration control in this paper. In component level isolation, a vibration isolation device provides articulation while reducing the vibration transmission between vibration source and parts, which needs to be vibration-free. Among others, the Stewart platform manipulator is a good candidate for high speed and six degree of freedom(DOF) motion control[2] finding its applications in the area of multi-axis vibration control of a structure even under a relatively large payload[1,2,4,5].
The vibration control application of a Stewart platform is quite different from using a Stewart platform as a flight simulator or as a multiple DOF parallel link manipulator. The stroke of actuators required for damping structural vibration is of the order of magnitude of microns, and frequency response performance should achieve the range of kHz. Force capability required by vibration control devices varies in different applications as per the requirement. On the other hand, the Stewart platform manipulator has a critical drawback, against the conventional vibration control devices, that the high nonlinearity and uncertainty in its dynamics. So, one of the open problems as mentioned by Dasgupta et. al.[3], study of dynamic behavior of the manipulator through extensive simulation and analytical/numerical tools for ODE system, is combined along with the application of a simple control strategy, for the vibration isolation application. This paper is organized as the following: In section II we would present why we use cubic configuration of Stewart platform for vibration isolation applications stating the advantages over conventional configurations. The section III carries the Newton-Euler’s closed-form dynamic formulation for the general Stewart platform and its application to vibration isolation problem. The results obtained by applying a simple PD control algorithm to the system is shown and discussed in section VI.

STEWART PLATFORM

A general Stewart platform, shown in figure 1, consists of six variable length actuators connecting the mobile plate to a base plate. As the length of actuators change, the mobile plate of the platform is able to move in all six DOF with respect to the base plate.

![Fig. 1: Stewart platform](image1)

![Fig. 2: Cubic configuration](image2)

This mechanism distinguishes themselves from other multiple DOF motion generators in that all actuators are linear motion actuators. One of the important properties of the Stewart platform for vibration control application is that, if the axial forces can be
measured and eliminated, all the forces and hence all of the vibration created by these forces can be eliminated because there is only axial forces being transferred from base to mobile plate. They can be designed to carry large loads and remain stable in unpowered configuration.

**Cubic Configuration**

The primary difficulty with general Stewart platform is that motions are strongly coupled and the motion in any Cartesian direction requires motion of all the legs, resulting in mathematical complexity of control design. So, “Cubic configuration” [1] was proposed for vibration control applications. As in the figure 2, the vertices A12, A34, and A56 forms one plane and B16, B23 and B45 form a second plane, forming the base plate and mobile plate of Cubic Stewart platform, where the connections between plates forms the six legs.

The cubic configuration has several unique features because of which it was used in many vibration isolation applications[1,4,5]. A few to mention, the orthogonality of adjacent legs guarantees the motion of mobile plate to be controlled independently by pair of actuators. It facilitates the utilization of SISO control algorithm for multiple DOF active vibration isolation problems. It has maximum uniformity of control authority in all directions and simplifies the kinematic relationship between the motion of each actuator and that of the mobile plate.

**NEWTON-EULER’S FORMATION**

The displacement, acceleration and force output, at the mobile plate caused by the disturbance is measured and is fed to the controller. The controller generates the control signal and is fed to the actuators of the each leg. The actuators generate counter vibration forces and the mobile plate is stabilized. To study the cause and effect of the disturbance and the control forces respectively, a thorough knowledge of the kinematics and dynamics of the Stewart platform is needed.

The dynamic formulation[6] and derivation of dynamic equations for parallel manipulators is quite complicated, because of their closed-loop structure and kinematic constraints. Euler-Lagrange’s formulation results in a system of differential algebraic equations, which are quite complicated to solve and leads to large symbolic calculations to find partial derivatives. The Newton-Euler’s formulation requires no evaluation of derivatives and so obviated a lot of cumbersome calculations. B. Dasgupta et. al.[6] have adopted the Newton-Euler approach for developing the closed-form dynamic equations of the Stewart platform, which are essential for forward dynamics and control system design. The general Stewart platform has a base and a platform connected by six extendable legs connected through spherical joint (6-SPS), or a spherical joint at one end
and a universal joint (6-UPS) at the other. The dynamic equations of the 6-UPS Stewart platform alone are used for vibration isolation application in this paper.

The kinematics and dynamics of one leg was considered and the expression for the constraint force at the spherical joint at the top of leg was derived. Then complete system of dynamic equations was obtained by considering kinematics and dynamics of the platform.

**Kinematics of the legs of Stewart platform**

**Position Analysis.** Vector loop equation from figure 3, is written as

\[ S = q + t - b \]  
(1)

Length of leg

\[ L = |S| \]  
(2)

**Velocity Analysis.** Velocity of platform point

\[ \dot{S} = \omega \times q + \dot{q} \]  
(3)

Sliding velocity between two parts of leg

\[ \dot{L} = s \cdot \dot{S} \]  
(4)

**Acceleration Analysis.** Acceleration of the platform connection point

\[ \ddot{S} = \ddot{q} + \alpha \times q + \omega \times (\omega \times q) \]  
(5)
Dynamic Analysis of legs

The general expression for all the legs

\[ F_s = Q_i \ddot{q}_i - \dot{Q}_i \ddot{q}_i \alpha + V_i - s_i F_i \]  \hspace{1cm} (6)

Kinematics and Dynamics of movable platform

Position vector of the center of gravity of the platform expressed in the base frame would be obtained as

\[ \vec{R} = r \vec{R}_q \]  \hspace{1cm} (7)

Solving the task space dynamics equations we get the equation of motion as

\[ J \begin{bmatrix} \ddot{\vec{i}} \\ \dot{\alpha} \end{bmatrix} + \eta = HF + \begin{bmatrix} \mathbb{R} F_{ext} \\ \mathbb{R} M_{ext} \end{bmatrix} \]  \hspace{1cm} (8)

Fext and Mext are the external force and external moment (disturbance) to be controlled. The derivation for Newton-Euler formulation is given in detail in reference[6].

CONTROL LAW

To generate counter vibratory forces a simple PD control algorithm is used, using the expression given below for the task-space. For a sine wave input the response of the system is illustrated in figure 4. The responses are for the Z direction, with and without control.

\[ F_{task} = \text{diag}[K_{p1}, K_{p2}, K_{p3}](t_0-t) + \text{diag}[K_{v1}, K_{v2}, K_{v3}](dt_0/dt) \]  \hspace{1cm} (9)

\[ M_{task} = \text{diag}[K_{p4}, K_{p5}, K_{p6}](\theta_0 - \theta) + \text{diag}[K_{v1}, K_{v2}, K_{v3}](\dot{\omega}_0 - \dot{\omega}) \]  \hspace{1cm} (10)

SUMMARY

A suitable configuration for the active vibration isolation application is identified and the forward dynamics of the Stewart platform is studied and applied for a vibration isolation problem, using the Newton-Euler’s formulation. The formulation is implemented using a MATLAB routine and one set of simulation results is shown as illustrations. A simple PD control law for active vibration isolation is developed using the position and velocity of the system, and is found to be effective.
Fig. 4: Response of the system with and without control, in z direction
REFERENCE


