Analysis Of Independent Suspension Linkages

Madhusudan Raghavan

Powertrain Systems Research Lab
General Motors R&D Center
Warren, Michigan

ABSTRACT

We present algorithms for determining the attachment point locations of various spatial in-parallel mechanisms to achieve desired motion of the coupler element. This is done for mechanisms with a variety of component chains such as R-S, S-P, S-S. The algorithms formulate the various problems as sets of algebraic equations to be solved for the attachment point locations. These procedures can be utilized to generate good starting points for detailed elastokinematic system optimization.

INTRODUCTION

The proper design of automotive suspensions is critical for achieving good “ride and handling.” Ride refers to the vehicle’s behavior during straight-line motion. Ride metrics include measures of how well the suspension absorbs bumps on the road and how the vehicle behaves during acceleration and braking. Handling refers to the vehicle’s response to steering inputs. Over the years a variety of suspension mechanism types have been used in automobiles. These include the short-long arm suspension [1], the MacPherson strut [2], the multi-link rear suspension [3, 4], the multi-link front suspension, and the short-long arm front suspension with a true king-pin [5], etc.

Numerous researchers have studied computer-based analysis and synthesis of automotive suspensions. Suh [1] uses displacement matrices coupled with constraint equations of links to give practical kinematic equations for analysis and synthesis of three dimensional suspension linkages. Suh [6], has also developed the notion of instantaneous screw axes for the vehicle sprung mass. Iijima et al. [7] describe the issues involved in the design of the front and rear suspensions for the Nissan 300ZX. Fuhrmann [8] describes the Porsche chassis design philosophy and in particular, the elastokinematic design of the Weissach-Axle on the Porsche 928. Simionescu, Smith and Tempea [9] develop a kinematic model of a rack-and-pinion type steering linkage and then perform mechanism synthesis to ensure Ackermann steering and good transmissibility of motion. Dijksman et al., [10] show by means of detailed mathematical analysis that specific six-bar steering linkages of the Watt-II type are better approximations to the Ackermann steering requirements than four-bar linkages.

SIGNIFICANCE OF THIS PAPER

The present work is a compilation of various mathematical models for rapidly computing suspension linkage geometry parameters from ride-handling requirements. These procedures can be used to generate good starting points for detailed elastokinematic system optimization.

EXPRESSING REQUIREMENTS MATHEMATICALLY

Requirements on wheel motion relative to the sprung mass have traditionally been expressed as planar quantities, such as the desired suspension linkage instant center locations in vehicle front- and side-view, ride camber, ride tread, ride caster, ride fore-aft, etc. (Please refer to SAE publication on vehicle dynamics terminology [11], for a description of parameters such as caster, camber, etc.). Let the location of the front- and side-view instant centers be respectively, $\begin{pmatrix} fvic\_x \\ fvic\_y \\ fvic\_z \end{pmatrix}$ and $\begin{pmatrix} svic\_x \\ svic\_y \\ svic\_z \end{pmatrix}$, in a coordinate system Σ fixed to the sprung mass. Let the wheel center location be represented by the vector $\begin{pmatrix} x9 \\ y9 \\ z9 \end{pmatrix}$. The swing center coordinates may be related to ride camber, ride tread, ride fore-aft, and ride caster, by the following equations

$$ride\,camber = \frac{1.0}{y9 - fvic\_y}\,rtod, \qquad ride\,tread = -\frac{z9 - fvic\_z}{y9 - fvic\_y}, \tag{1}$$

where $rtod$ is the conversion factor to go from radians to degrees. Similarly,

$$ride\,caster = -\frac{1.0}{x9 - svic\_x}\,rtod, \qquad ride\ fore-aft = -\frac{z9 - svic\_z}{x9 - svic\_x} \tag{2}$$

Let R (3 by 3 matrix) and d (3 by 1 vector) represent respectively, the orientation and position of the wheel coordinate system E relative to Σ . Then one may show that:

$$\frac{\partial d}{\partial z} = \begin{pmatrix} ride\ fore-aft \\ ride\,tread \\ 1.0 \end{pmatrix} \tag{3}$$

Correspondingly $\frac{\partial R}{\partial z}$, the rate of change of wheel orientation with jounce-rebound, may be obtained using the following relationships.

Let $R = \begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{pmatrix}$,

where α, β, γ , are yaw, pitch, and roll parameters respectively. Then

$\frac{\partial R}{\partial z} = \begin{pmatrix} 0 & -\frac{\partial\alpha}{\partial z} & \frac{\partial\beta}{\partial z} \\ \frac{\partial\alpha}{\partial z} & 0 & -\frac{\partial\gamma}{\partial z} \\ -\frac{\partial\beta}{\partial z} & \frac{\partial\gamma}{\partial z} & 0 \end{pmatrix}$ at zero jounce, when α, β, γ are zero. The rates of change of α, β, γ with jounce-rebound are related to suspension geometric parameters as follows.

$$A\begin{pmatrix} \frac{\partial\alpha}{\partial z} \\ \frac{\partial\beta}{\partial z} \\ \frac{\partial\gamma}{\partial z} \end{pmatrix} = b\,, \tag{4}$$

$$\text{where } A = \begin{pmatrix} -\cos(toe)\cos(camber) & \sin(camber) & 0 \\ 0 & \sin(toe)\cos(camber) & \cos(toe)\cos(camber) \\ \tan(kpi) & -\frac{1}{\cos^2(caster)} & \tan(kpi)\tan(caster) \end{pmatrix}$$

$$b = \begin{pmatrix} -\cos(toe)\cos(camber)\frac{\partial toe}{\partial z}dtor + \sin(toe)\sin(camber)\frac{\partial camber}{\partial z}dtor \\ \cos(camber)\frac{\partial camber}{\partial z}dtor \\ -\frac{1.0}{\cos^2(caster)}\frac{\partial caster}{\partial z}dtor \end{pmatrix}.$$

CONSTRAINT EQUATIONS FOR VARIOUS SUSPENSION LINKAGES

All independent suspension linkages are typically comprised of a knuckle (the coupler) and several chains of links and joints in-parallel between the knuckle and the sprung mass. For the SLA suspension linkage, there are 3 such chains of links and joints, viz., 2 R-S chains (the control arms), and 1 S-S chain (the tie-rod). The position and velocity constraint equations for the various chains are as follows.

R-S chain

Position constraints:

$$(Rp + d - f)\bullet(Rp + d - f) = r^2 \tag{5}$$

$$(Rp + d - f)\bullet v = 0 \tag{6}$$

Velocity Constraints:

$$\left(\frac{\partial R}{\partial z}p + \frac{\partial d}{\partial z}\right)\bullet(Rp + d - f) = 0 \tag{7}$$

$$\left(\frac{\partial R}{\partial z}p+\frac{\partial d}{\partial z}\right)\bullet v=0 \tag{8}$$

where d and R represent respectively, the position vector and orientation matrix of coordinate system E (the coordinate system on the knuckle) in Σ, p is the position vector of the control-arm outer ball-joint in E, f is a vector from Σ to the inner hinge joint of the revolute joint, r is the length of the control-arm, and v is a unit vector in Σ along the revolute joint axis. The partial derivatives of R and d with respect to z ,represent rates of change of orientation and position of the wheel during jounce-rebound.

<u>S-S chain</u>

Position constraint:

$$(Rp+d-f)\bullet(Rp+d-f)=r^2 \tag{9}$$

Velocity Constraint:

$$\left(\frac{\partial R}{\partial z}p+\frac{\partial d}{\partial z}\right)\bullet(Rp+d-f)=0 \tag{10}$$

where the various parameters are as defined above, except that f is the position vector in Σ of the inner ball-joint.

The MacPherson strut suspension is comprised of a strut portion above which is a S-P chain from the sprung mass to the knuckle, and a R-S chain below, similar to the SLA suspension lower control arm. The tie-rod (or toe-link) is represented by a S-S chain as before. The position constraint equations for the S-P chain are as follows:

<u>S-P chain</u>

Position constraints:

$$\begin{pmatrix}a_1 & 0 & c_1\end{pmatrix}\bullet\left(R^T\begin{pmatrix}X_P\\Y_P\\Z_P\end{pmatrix}-R^Td\right)+1=0 \tag{11}$$

$$\begin{pmatrix}0 & b_2 & c_2\end{pmatrix}\bullet\left(R^T\begin{pmatrix}X_P\\Y_P\\Z_P\end{pmatrix}-R^Td\right)+1=0 \tag{12}$$

Velocity constraints:

$$\begin{pmatrix}a_1 & 0 & c_1\end{pmatrix}\bullet\left(\frac{\partial R^T}{\partial z}\begin{pmatrix}X_P\\Y_P\\Z_P\end{pmatrix}-\frac{\partial R^T}{\partial z}d-R^T\frac{\partial d}{\partial z}\right)=0 \tag{13}$$

$$\begin{pmatrix}0 & b_2 & c_2\end{pmatrix}\bullet\left(\frac{\partial R^T}{\partial z}\begin{pmatrix}X_P\\Y_P\\Z_P\end{pmatrix}-\frac{\partial R^T}{\partial z}d-R^T\frac{\partial d}{\partial z}\right)=0 \tag{14}$$

where the parameters a_1, c_1, b_2, c_2 define the line of slide of the prismatic joint in E via the following equations:

$$\begin{pmatrix} a_1 & 0 & c_1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + 1 = 0, \qquad \begin{pmatrix} 0 & b_2 & c_2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + 1 = 0 \tag{15}$$

The synthesis equations for the lower control-arm (R-S chain) and the tie-rod (S-S) chain are the same as for the SLA suspension (i.e., Eqs. (5) – (10)). Multi-link suspension linkages typically have 3-5 S-S chains connected in parallel between the sprung mass and the knuckle with possibly a R-S chain thrown in when necessary to get an adequate number of constraints. The position and velocity constraint equations for these chains have all been discussed above. Raghavan [12] describes the synthesis of the various types of kinematic chains (R-S, S-P, and S-S) for planar situations and finitely separated synthesis positions. We suggest that specifying one design position (when the wheel is at zero jounce) and the rates of change of the various wheel kinematic parameters at that design position, is a good approach to identifying compact designs.

SYNTHESIS FOR LINEAR TOE CURVES AND ROLL CENTER HEIGHT

The tie-rod is modeled as a solid bar with a ball-joint at either end. For the present section, we may assume that the rest of the suspension mechanism (i.e., everything except the tie-rod) has already been synthesized. We move the wheel and this "incomplete" suspension (i.e., suspension minus tie-rod) through the range of jounce-rebound motion by means of an ADAMS-type analysis. The tie-rod is then synthesized using three finitely separated design positions. For mathematical details and illustrative examples, please refer to the paper by Raghavan [13].

The roll center is considered an important factor in determining overall vehicle ride and handling quality. It is the point (in front-view) at which the line from the suspension instant center to the tire contact patch intersects the central plane. From the standpoint of vehicle dynamics, the roll center is the point in front view, about which the sprung mass rolls when the vehicle goes into a turn. Raghavan [14] has developed a constraint equation specifying the relative lengths of the control arms for prescribed roll center height change.

SYNTHESIS FOR PRESCRIBED ACKERMANN ERRORS

In the traditional synthesis approach, the steering linkage of Figure 1(a) is approximated as a planar four-bar (Figure 1(b)) and synthesized for perfect Ackermann conditions by laying out two construction lines from the kingpin axes to the center of the rear axle, as shown in Figure 2. The ball joints connecting the steer-arms to the tie-rods are selected on these construction lines. The resulting design is accurate in meeting the perfect Ackermann steer requirement through about 20 degrees of steer. The traditional procedure is extended here to enable synthesis for prescribed values of Ackermann error because most real-world designs have some amount of Ackermann error deliberately designed into them to account for tire slip angles, which distort the so-called perfect

Ackermann geometry. Consider the vector chain in Figure 3 representing the four-bar linkage of Figure 1(b). The horizontal and vertical components of the vector loop equation representing the four-bar linkage are (from Ref. 15):

$$r_1 \cos\theta_1 + r_2 \cos\theta_2 + r_3 \cos\theta_3 + r_4 \cos\theta_4 = 0, \tag{16}$$

$$r_1 \sin\theta_1 + r_2 \sin\theta_2 + r_3 \sin\theta_3 + r_4 \sin\theta_4 = 0. \tag{17}$$

After squaring and adding to eliminate θ_3, and rearranging terms we get

$$K_1 \cos\theta_2 + K_2 \cos\theta_4 + K_3 = \cos(\theta_2 - \theta_4), \tag{18}$$

where $K_1 = \frac{r_1}{r_4}, K_2 = \frac{r_1}{r_2}, K_3 = \frac{r_3^2 - r_1^2 - r_2^2 - r_4^2}{2 r_2 r_4}$. As the linkage is symmetric we may write

$$\theta_{2_0} + \theta_{4_0} = 360 \tag{19}$$

where the subscript 0 represents the straight-ahead or zero-steer position. Suppose that the vehicle makes a left turn. The left wheel (in this case, the inner wheel) is steered inwards by $\Delta\theta_2$ degrees and the right wheel (the outer wheel) is steered by a corresponding amount, say $\Delta\theta_4$. Using Eq (19) this may be represented as

$$K_1 \cos(\theta_{2_0} + \Delta\theta_2) + K_2 \cos(\theta_{4_0} + \Delta\theta_4) + K_3 = \cos(\theta_{2_0} + \Delta\theta_2 - \theta_{4_0} - \Delta\theta_4), \tag{20}$$

Eq. (20) serves as our synthesis equation. Since the design is independent of linkage scale, we may set $r_1 = 1.0$ and let $r_2 = r_4 = 0.2$.

$$r_3 = 1 - 2(0.2)\cos\theta_{4_0} \tag{21}$$

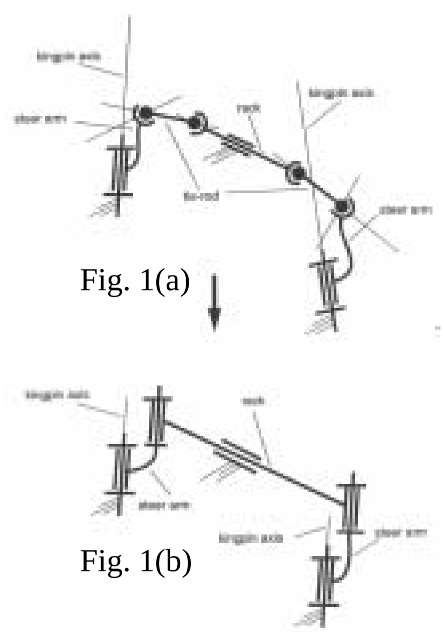

Fig. 1: Steering Linkage

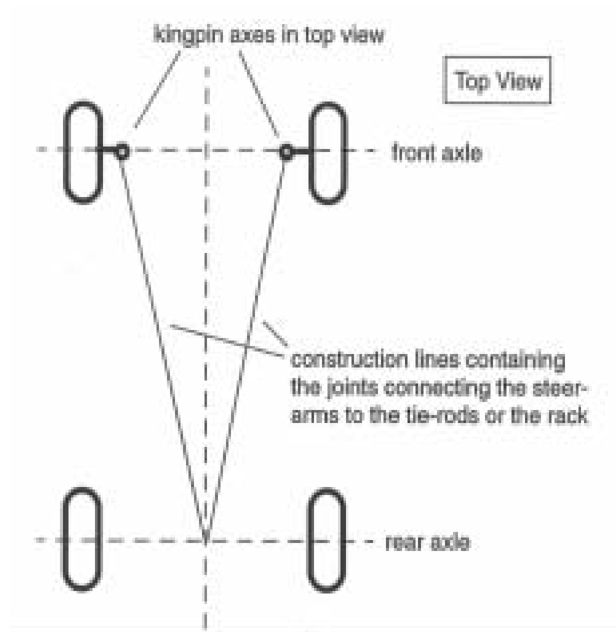

Fig. 2: Construction lines for Ackermann steer

Substituting the above values and expressions for r_1, r_2, r_3, r_4 into Eq. (20) and using Eq. (19) to eliminate θ_{2_0} from the resulting expression we obtain an equation $f(\theta_{4_0}, \Delta\theta_2, \Delta\theta_4) = 0$, relating $\theta_{4_0}, \Delta\theta_2, \Delta\theta_4$. This may be used as follows. For a given inner steer angle $\Delta\theta_2$ we compute the corresponding value of the outer steering wheel angle $\Delta\theta_4$, using the following equation (the mathematical definition of Ackermann error)

$$\Delta\theta_4 = \text{ackermann error} + a\tan\left(\frac{1.0}{\cot(\Delta\theta_2) + \dfrac{trackwidth}{wheelbase}}\right)$$

Substituting these values of $\Delta\theta_2$ and $\Delta\theta_4$ in the equation $f(\theta_{4_0}, \Delta\theta_2, \Delta\theta_4) = 0$, we may solve for $\Delta\theta_{4_0}$ to determine the inclinations of the steer-arms at the zero steer position for a prescribed Ackermann error.

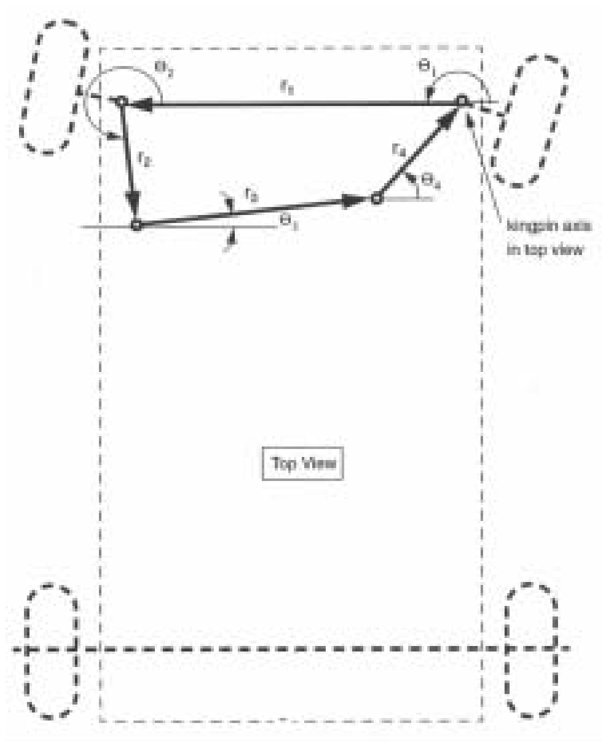

Fig. 3: Steering linkage as a planar 4-bar

USEFULNESS & SUMMARY

Algebraic models for representing various suspension linkage types have been presented. They can be utilized as a synthesis tool to generate good starting points for detailed elastokinematic system optimization.

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