Simulation Of Software For Four-Bar Function Generator Mechanism

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ABSTRACT

This is an effort to develop an interactive software for structural synthesis of four bar function generator mechanism. The steps involved in the method of structural synthesis based on "Relative Pole Method" & Freudenstein's Displacement Equation" for 3-precision points [1,2] have been recast in a format suitable for implementation on a digital computer. The methodology thus evolved has been combined with the algebraic procedures for structural analysis [3,4,5] to develop a unified computer program for structural synthesis and analysis of simple 4-bar Function Generator for algebraic functions like Y=1/X, X^2, X^3, ln(X), log(X), exp(X) etc. Application of this program are presented, the inputs to the program are the function to be generated, lengths of two links, (frame and crank), initial position of crank and range of motion of crank and rocker. The output of program consists of dimensions of other two links, tabulated values of percentage error, value of Y by function and by calculation of link lengths, mechanical advantage, type of mechanisms (crank rocker, double crank, double rocker and also the position of link where its gets jammed at certain position in some cases), Grashof's condition.

This software can be utilized for synthesis of complex higher mechanisms by using Transformation of Binary Links. It can also be used as a tool in preliminary design stage for analysing and modifying the behavior of proposed mechanism. This software has been successfully used for explaining, teaching as well as experimentations for UG courses of Kinematics of Machines.

1. INTRODUCTION

Structural synthesis and analysis of plane linkages and other mechanisms has been the subject of several investigations [6,7] . The methods employed for structural synthesis of kinematic chains in the earlier methods were based on pure intuition and inspection graph theory and transformation of multiple jointed chains , referred to as binary chains . The element of visual inspection is present in all these studies with the result that the possibility of human error affecting the results cannot be ruled out. There is thus a need for developing a fully computerised method for structural synthesis which eliminates this error and further enables the designer to obtain results quickly and easily. The aim of this paper presentation is to fulfill such a need. The method of Freudenstein's Displacement Equation [1] (analytical method) and Relative Pole Method , ie. one of the graphical method of kinematic synthesis [2] has been taken up for computerisation . The computerised methodology presented here is applicable to all types of functions. Further the approach employed in the computerised method presented here is aimed to present the mechanism along with the animation and followed by the calculation of mechanical advantage, structural error and percentage structural error at every position of mechanism and identification of the type of mechanism ie. Crank - crank, crank- rocker , rocker- rocker [2,3,4] jammed position of input crank[5].

Chebyshev determines that the best linkage approximation to a function occurs when the absolute value of the maximum structural error between the precision points & at both ends of the range are equalized [1]. Chebyshev spacing of precision points is employed to minimise the strucural error. And also the Freudeustein's Equation Technique used for dimensional synthesis will also give the minimised structural error.

2. MEHODOLOGIES USED FOR FOUR BAR SYNTHESIS

The two methods used [1,2] for the synthesis of four bar function generator are :

2.1 RELATIVE POLE METHOD (GRAPHICAL METHOD)

The synthesis of linkages involves moving of a link from a first position to several others , sometimes under specification of velocity & acceleration[1,2]. Here the aim is to synthesise the linkage without imposing specifications with regards to velocity and acceleration . The pole is defined as a point about which the link rotates through a finite angle with unique translation or rotation.

2.2 FREUDENSTEIN'S DISPLACEMENT EQUATION TECNIQUE. (AN ALGEBRAIC METHOD)

This algebraic method utilises Freudenstein's displacement equations for 3-precision point function generation[1,2]. This technique has been extended to 4 & 5

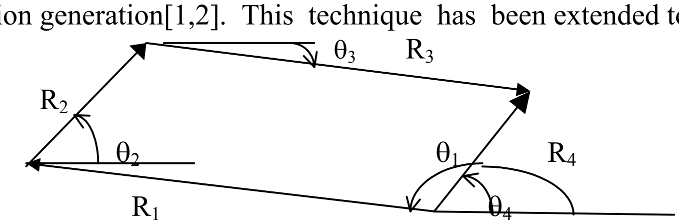


Fig 1.

precision points and by regarding the scale factors R_ϕ & R_Ψ of input & output rotations as unknowns. It has been extended to other linkages but these cases are not presented here.The equation can be derived from the loop-closure equation for the Fig 1.

$$R_1 + R_2 + R_3 - R_4 = 0 \qquad \text{Eq.(1)}$$

If this complex equation is separated into real and imaginary components, two algebraic equations are produced :

$$R_1 \cos\theta_1 + R_2 \cos\theta_2 + R_3 \cos\theta_3 - R_4 \cos\theta_4 = 0 \qquad \text{Eq.(2)}$$

$$R_1 \sin\theta_1 + R_2 \sin\theta_2 + R_3 \sin\theta_3 - R_4 \sin\theta_4 = 0 \qquad \text{Eq.(3)}$$

Assuming that the ground link is along the X-axis $\theta_1=180°$, to synthesize a function generator , θ_3 is not of interest and hence eliminating θ_3 ,

$$(R_3^2 - R_1^2 - R_2^2 - R_4^2) / 2R_2 R_4 + (R_1 \cos\theta_2) / R_4 - (R_1 \cos\theta_4) / R_2 = \cos(\theta_2-\theta_4) \qquad \text{Eq.(4)}$$

In a more compact form , Freudenstein's. equation can be given as

$$K_1 \cos\theta_2 + K_2 \cos\theta_4 + K_3 = -\cos(\theta_2-\theta_4) \qquad \text{Eq.(5)}$$

where $K_1 = R_1 / R_4$; $K_2 = - R_1 / R_2$; $K_3 = (R_3^2 - R_1^2 - R_2^2 - R_4^2)/ 2 R_2 R_4$ Eq.(6)

Notice that the K's are 3 independent algebraic expressions counting the 3 unknown lengths of the links. Freudenstein's equation is the displacement for the four bar linkage which holds true for each position of the linkage. Thus for 3 prescribed positions , the equation can be written for each position. The notation will be changed at this point to avoid double subscribes the 3 angles for the 3 prescribed positions of R_2 with respect to the fixed axis will be ϕ_1, ϕ_2 and ϕ_3 while those of R_4 will be Ψ_1 , Ψ_2 and Ψ_3 as in Fig. 2

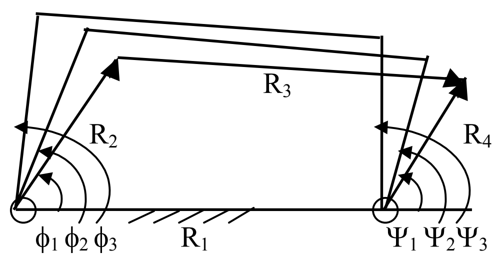


Fig 2.

Thus Freudenstein's equation for 3 prescribed positions is:

$$K_1 \cos\phi_1 + K_2 \cos\Psi_1 + K_3 = -\cos(\phi_1 - \Psi_1) \qquad \text{Eq.(7)}$$

$$K_1 \cos\phi_2 + K_2 \cos\Psi_2 + K_3 = -\cos(\phi_2-\Psi_2) \qquad \text{Eq.(8)}$$

$$K_1 \cos\phi_3 + K_2 \cos\Psi_3 + K_3 = -\cos(\phi_3-\Psi_3) \qquad \text{Eq.(9)}$$

By applying the Cramer's rule the Eqs.(7)-(9) can be solved and given the length R_1, solving the resulting system of two equations for K_1 and K_2.

$$K_1 = (w_3 w_5 - w_2 w_6) / (w_1 w_5 - w_2 w_4) \qquad \text{Eq.(10)}$$

$$K_2 = (w_1 w_6 - w_3 w_4) / (w_1 w_5 - w_2 w_4) \qquad \text{Eq.(11)}$$

In which ,$w_1=\cos(\phi_1)-\cos(\phi_2)$; $w_2=\cos(\Psi_1)-\cos(\Psi_2)$; $w_3=-\cos(\phi_1-\Psi_1)+\cos(\phi_2-\Psi_2)$;

$w_4=\cos(\phi_1)-\cos(\phi_3)$; $w_5=\cos(\Psi_1)-\cos(\Psi_3)$; $w_6=-\cos(\phi_1-\Psi_1)+\cos(\phi_3-\Psi_3)$.

$$K_3 = -\cos(\phi i - \Psi i) - K_1 \cos\phi i - K_2 \cos\Psi i \qquad (\text{ where } i = 1,2 \text{ or } 3) \qquad \text{Eq.(12)}$$

The link lengths may be expressed in terms of the known K's by using equation

$$R_4 = R_1 / K_1 \; ; \quad R_2 = R_2 / K_2 \; ; \quad R_3 = \left(2\, K_3\, R_2\, R_4 + R_1 + R_4 \right)^{1/2} \qquad \text{Eq.(13)}$$

3. STRUCTURAL ANALYSIS OF THE FOUR BAR MECHANISM.

In the synthesised four bar function generator the following analysis is done.

3.1 Checking the satisfaction of Grashof's critrion [4].

3.2 To identify the kind of linkage ie.Doublecrank, crankrocker,doublerocker mechanism [1,2], so that as per the application selection of type of mechanism will be easier.

3.3 The linkage is also made to run through its range and its performance in terms of structural error , percentage structural error , mechanical advantage is found for different positions of input link [1,2]

3.4 A planar four bar linkage has only one degree of freedom . When additional degrees of freedom are needed, more links may be added and so an N- bar linkage is formed. With multiple degrees of freedom, an N- bar linkage requires multiple inputs. If these inputs are independent , the linkage may function as a programmable linkage or a robot. If these inputs are coupled , the linkage may become a single degree of freedom mechanism, which still provides more versatility and design opportunity over a four bar linkage. Hence it is important to know the rivolvability or mobility of the linkage [3].

3.5 The extreme velocity ratio is found at which the mechanism is jammed [5]

3.6 The program also calculates the rms (root mean square) value of error.

4. Summary of output (of computer program [4.2]) for various functions for the same input values

The various inputs are:

Length of the input link , O_aO_b = 100mm;

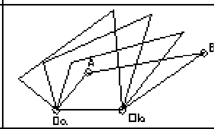
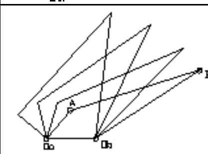
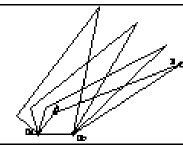
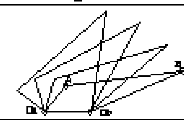
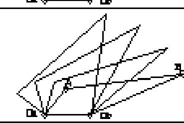
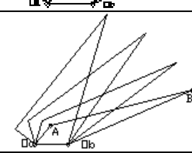
Length of the crank , O_aA=75mm;

Initial position of crank,a input link ϕ_s= 50°;

Range of variation of crank, , $\Delta\phi$ = 90° ;

Range of variation of output link , $\Delta\Psi$ = 60°

Table 1: The overall output of the program for various functions are

Function	R_3 (AB)	R_4 (ObB)	RMS	Type of mechanism	Grashof's Criteria	Synthesised Mechanism
$Y=1/X$[+]	179.9	152.5	0.01793	CR Type	Not Satisfied	
$Y=X^2$	278.5	254.8	0.05248	CR Type	Satisfied	
$Y=X^3$	385.5	363.1	0.23639	CR Type	Satisfied	
Y=ln(X)	196.1	169.8	0.01119	CR Type	Not Satisfied	
Y=log(X)	196.1	169.8	0.00486	CR Type	No	
Y=exp(X)	330.9	307.8	0.12067	CR Type	Satisfied	

4.1 Synthesis of the four bar function generation mechanism for the function y=1/x :

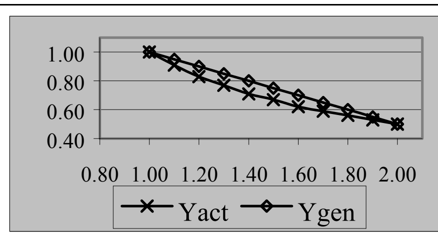
<table>
<tr><th>Inputs to the program</th><th>Output in terms of lengths</th><th>Fig.3 Comparison of actual & generated, Y=1/X function</th></tr>
<tr><td>O_aO_b= 100mm
O_aA =75mm
ϕ_s = 50°
$\Delta\phi$ = 90°
$\Delta\Psi$ = 60°</td><td>By analytical method
AB=178.58 mm;
O_bB = 153.56mm
By graphical method
AB=179.86 mm;
O_bB = 152.47mm</td><td>
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Table 2. Output in the form of tabulated results

[+] The detailed sample output has been shown in the section 4.1

Xact	Yact	Fi(ϕ)	Si (Ψ)	Ygen	error	Percentage error	Mechanical advantage
1.00	1.00	50.00	31.40	1.00	0.00	0.00	1.21
1.10	0.91	59.00	37.40	0.95	0.04	4.50	1.24
1.20	0.83	68.00	43.40	0.90	0.07	8.00	1.27
1.30	0.77	77.00	49.40	0.85	0.08	10.50	1.31
1.40	0.71	86.00	55.40	0.80	0.09	12.00	1.36
1.50	0.67	95.00	61.40	0.75	0.08	12.50	1.41
1.60	0.62	104.00	67.40	0.70	0.07	12.00	1.47
1.70	0.59	113.00	73.40	0.65	0.06	10.50	1.54
1.80	0.56	122.00	79.40	0.60	0.04	8.00	1.62
1.90	0.53	131.00	85.40	0.55	0.02	4.50	1.70
2.00	0.50	140.00	91.40	0.50	0.00	0.00	1.79

Calculated RMS value is 0.01793

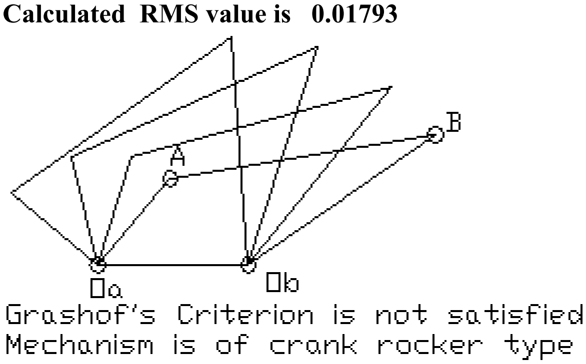

Fig.4. Synthesised four bar function generator

COMMENTS: The above sample output is for the function y=1/x, with the initial positions 50^0 and 31.4^0 of the crank and rocker respectively will result into a mechanism which will not satisfy the Grashof's criterion and it is of crank rocker type of mechanism. The comparison between Generated function and the actual output function is shown in fig.3. The error at different crank positions is given in Table No.2

4.2 COMPUTER PROGRAM

A unified computer program was developed using the methodologies presented in section [2,2.1,2.2] for the kinematic synthesis of four bar function generator for the various functions ie. Y=1/X , X^2 , X^3 , ln (X) , log (X) , exp(X) along with the algebraic procedures presented in the section 4 for analysing the linkage synthesised along with the animation . Given the input of $\Delta\phi$ & $\Delta\psi$, input link (crank) length , fixed link length & ϕ_S the program in 'C' language first calculates & prints the link lengths followed by the animation of mechanism for the results of analysis presented in section 3,3.1-3.6. The algorithm of the program is as follows :

Step 1 : Read the input variables

Step 2 : All initial calculations are done including precision points.

Step 3 : Find the coordinates of the 4-bar linkage by the analytical or graphical method or by both the methods.

Step 4 : Find the link lengths of the mechanism.

Step 5 : Find the error followed by the RMS value

Step 6 : Find the minimum value of RMS & select that mechanism .

Step 7 : Find the mechanical advantage, percentage of error & Tabulate the values

Step 8 : For few cases where crank doesn't make complete revolution locate the jammed position or last point upto which the mechanism moves.

Step 9 : Plot the mechanism / linkage

Step10: Check the satisfaction of Grashof's Criterion

Step11: Identification of the types of the mechanism i.e. double crank, double rocker, crank rocker, output link is the shortest by animating the mechanism for the given input angles.

5. CONCLUSION:

Using the Relative Pole Method and Freudenstein's Displacement Method , the mechanism can be plotted and animated s for the input range will assist the designer for faster selection of 4 bar mechanism as it is the basic linkage required for all the applications. The analysis results will guide the designer to decide about the constraints to be put on mechanism. The computer program can be used in iterative manner by varying the initial input parameters , viz frame length (OaOb) , crank length (OaA) , initial position of crank (ϕ_S), range of variation of input crank ($\Delta\phi$) & output link ($\Delta\psi$) . The error table is calculated for each run and RMS error is estimated thereof. By setting the cutoff value of permissible error, the optimised output may be obtained.

NOMENCLATURE

GC : Grashof's Criteria

RMS : Root mean square value

R_1 (OaOb) : Length of frame. (Millimeters)

R_2 (OaA) : Length of input link (crank length). (Millimeters)

R_3 (AB) : Length of Coupler link. (Millimeters)

R_4 (ObB) : Length of output link (rocker) (Millimeters)

Yact : Actual function value

Ygen : Generated function value

θ i : Inclination of the links in a mechanism with respect to X-axis. (degrees)

ϕ i : Position of crank with respect to frame (degrees)

Ψi : Position of output link with respect to frame (degrees)

ϕs : Initial position of crank with respect to frame (degrees)

$\Delta\phi$: Range of variation of crank motion expressed in angles (degrees)

$\Delta\Psi$: Range of variation of rocker motion expressed in angles (degrees)

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